# Improved Study of Vibrations of the Centrifuges with the Basket in Console

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*This paper approaches a field of advanced research: the study of vibrations of industrial centrifuges by applying the transfer matrix method. It will be established a study model considering the shaft as continuous medium. Bending vibrations coupled with axial vibrations will be studied. The transfer matrix and the transition matrices will be established. The case of the free vibrations will be developed.*

*Keywords: centrifuge, transfer matrix method, vibrations, natural frequencies, coupled vibrations.*

The centrifuge is a mechanical system composed of a flexible shaft and a basket mounted on the shaft (fig. 1.a). This machine realizes the process named centrifugal action. This is a mechanical process, which allows the accelerated separation of the components of heterogeneous systems.

The centrifuges can be horizontal or vertical. In this paper the horizontal centrifuge is considered [1].

In some books there is not approach about centrifuge vibration [2]. Anyway the reserved chapter of the dynamic behaviour is small and the problem is approached through simplify methods. In the field of activities are only few paper works, or this aim is not about the dynamics of centrifuge machine. Some paper works treat the approximate and construction of the subassembly [3] and other refers to the separation process of blending stock [4], [5].

This study has as objective the improvement of the study of vibrations of the centrifuge with the basket in console. The model of the centrifuges in order to study the vibrations is flexible shaft considered to be a continuous medium positioned on two bearing blocks and a basket fixed on the shaft. The shaft can effect bending vibrations in a plane which contains the center of mass of the basket and the longitudinal axis of the shaft (fig. 1.b) and axial vibrations along this axis (fig. 1.c). By choosing the system of axes presented at figure 1, with the *Oy* axis along the shaft, bending vibrations will are in the *xOy* plane (fig. 1.b) and axial vibrations will are along the *Oy* axis (fig. 1.c).

For the study of the vibrations can be used finite element method or transfer matrix method. Recently, mesh-free methods were developed and applied to obtain the vibration responses of various structures [6 - 8]. In this paper the transfer matrix method will be used.

This paper shows the calculation of the natural frequencies of the centrifuge. By knowing these natural frequencies, and also the critical speed, it will be avoided the operation of the centrifuge at this speed or around this kind of speed [9 - 11].

## The Basket

Each centrifuge has a basket. It is considered that the basket is fixed on the shaft in the point *O* (fig. 2) [3].

In the case of the centrifuges, the basket can not be considered a thin disk and the fixing point of the basket can not be in the centre of mass of the basket [4].

The d'Alembert's principle is applied [12]. First time the reduction of the system of d'Alembert's fictitious forces will be done in the center of mass of the basket (fig. 2.a):

$$\tau_C^{in}\begin{cases}\vec{F}^{in} = -m\vec{a}_C\\ \vec{M}_C^{in} = -\dfrac{d\vec{K}_C}{dt}\end{cases}. \qquad (1)$$

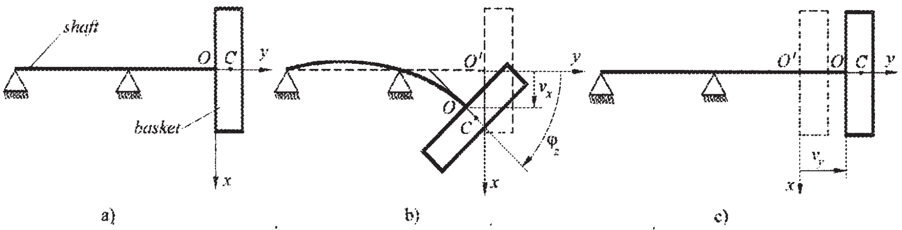


Fig. 1. Centrifuge with the basket in console

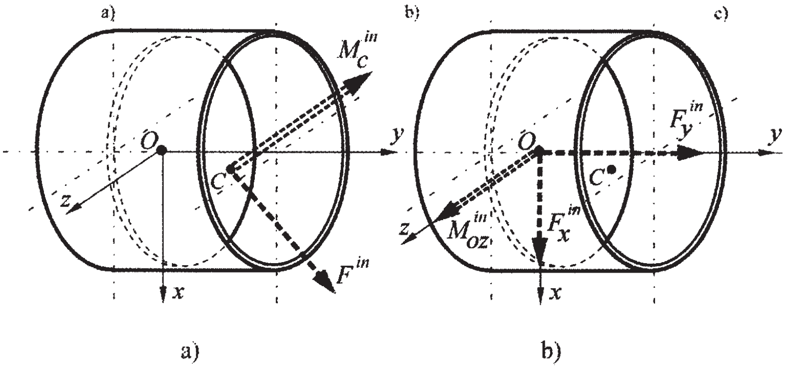


Fig. 2. The basket

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After, using the relation $\vec{M}_O^{in} = \vec{M}_C^{in} + \vec{r}_C \times \vec{F}^{in}$ ,the resultant moment of d'Alembert's fictitious forces is calculated with respect to the fixing point of the basket, *O*. The expressions for $\vec{F}^{in}$ and $\vec{M}_o^{in}$ are now:

$$\begin{cases} F_x^{in} = m\left(\omega^2 v_x + \varepsilon_z \eta + \omega_z^2 \xi\right) \\ F_y^{in} = m\left(\omega^2 v_y - \varepsilon_z \xi + \omega_z^2 \eta\right) \\ F_z^{in} = 0 \end{cases} \tag{2}$$

$$\begin{cases} M_{Ox}^{in} = \left(J_{Cxz} + m\xi\zeta\right)\varepsilon_z - \left(J_{Cyz} + m\eta\zeta\right)\omega_z^2 \\ M_{Oy}^{in} = \left(J_{Cyz} + m\eta\zeta\right)\varepsilon_z + \left(J_{Cxz} + m\xi\zeta\right)\omega_z^2 + m\zeta\omega^2 v_x \\ M_{Oz}^{in} = \left(J_{Cz}\omega^2 + m\xi^2\omega^2 + m\eta^2\omega^2\right)\varphi_z - m\eta\omega^2 v_x + m\xi\omega^2 v_y \end{cases} \tag{3}$$

where:

$$v_x = V_x(y)\cos(\omega t - \phi) \;;\; v_y = V_y(y)\cos(\omega t - \phi); \varphi_z = \frac{dv_x}{dy} = \Phi_z(y)\cos(\omega t - \phi);$$

$$\omega_z = \frac{d\varphi_z}{dt} = -\omega\Phi_z \sin(\omega t - \phi) \;;\; \varepsilon_z = \frac{d^2\varphi_z}{dt^2} = -\omega^2\varphi_z = -\omega^2\frac{dv_x}{dy}.$$

The uneven deposits of the operating material on the wall of the basket are the reason for which the centre of mass of the basket (including the operating material) is not placed on the revolution axis of the centrifuge. The bending vibrations in the plane that includes the centre of mass of the basket and the revolution axis will be studied ($\xi = 0$) (fig. 1.b).

Also, because of the high rigidity, the amplitude of the rotation of the cross section $\varphi_z$ has low values. $\omega_z^2$ has been neglected with respect $\varepsilon_z$. In these conditions the expressions of the resultant and the moment resultant of d'Alembert's fictitious forces can be written (fig. 2.b):

$$\begin{cases} F_x^{in} = m\omega^2 v_x - m\eta\omega^2\varphi_z \\ F_y^{in} = m\omega^2 v_y + m\xi\omega^2\varphi_z \\ F_z^{in} = 0 \end{cases} \tag{4}$$

$$\begin{cases} M_{Ox}^{in} = 0 \\ M_{Oy}^{in} = 0 \\ M_{Oz}^{in} = \left(J_{Cz}\omega^2 + m\xi^2\omega^2 + m\eta^2\omega^2\right)\varphi_z - m\eta\omega^2 v_x + m\xi\omega^2 v_y \end{cases} \tag{5}$$

In the expression (4) of $\vec{F}^{in}$ on the direction of the axial vibrations, *Oy* axis, appears the term $m\xi\omega^2\varphi$ that is a result of the bending vibrations. The bending vibrations excite axial vibrations. So, bending vibrations and axial vibrations must be studied together. These two types of vibrations have common natural frequencies.

## Hypothesis. Description of Method

*Hypothesis*

From the expressions of the resultant (4) and the moment resultant (5) of d'Alembert's fictitious forces it results that the axial vibrations and the bending vibrations are coupled. So it is considered that the shaft consists of straight elements of constant section which effect bending vibrations in a plane which contain the center of mass of the basket and the longitudinal axis of the shaft and axial vibrations along this axis.

It is considered that all the elements are compact and homogeneous. It is neglected the dimensions of the bearings.

It was constructed [13] a transfer matrix which keeps account of these types of vibrations.

## Description of method

The system shaft-basket is divided in straight elements which have constant section and which are not acted by concentrated forces or moments.

The elements obtained are numbered in order beginning with number *1* for the first element from the left side (fig. 3). The system will be traversed from the left to the right on the mean fibre.

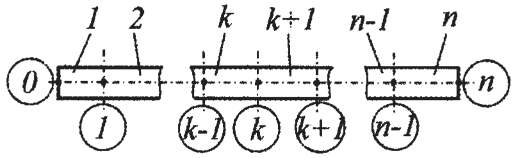


Fig. 3. Numbering of elements and nodes of the system

The points, where a saltus of section appears or concentrated forces or moments act, are named nodes. The numbering of the nodes begins with number *0* for the first node from the left side. So, the element number *k* will be bounded by the nodes *k-1*, at the left side, and *k* at the right side.

For each node *j* (*j=0,1,...,n*) is written a state vector (column matrix) $\{q\}_j$. The elements of these state vectors are proportionale to the displacement (deformation), rotation of the cross section, bending moment, shearing force, axial force.

For each element *k* (*k=1,2,... ,n* ) is written the transfer matrix $[A]_k$. The matrix relation for an element *k* is:

$$\{q\}_k = [A]_k \{q\}_{k-1}. \tag{6}$$

In the state vectors for the nodes from the ends of the system, *0* and *n*, will appear the boundary conditions. In the internal nodes, *1, 2,---, n-1*, can intervene: saltus of section, basket, elastic bearings. So, in a node *k* can be defined a state vector at the left $\{q\}_k^l$ and a state vector at the right $\{q\}_k^r$. The matrix relation in a node *k* can be written:

$$\{q\}_k^r = [H]_k \{q\}_k^l. \tag{7}$$

$[H]_k$ is the transit matrix in the node k and can be one of the matrices: saltus matrix, transition matrix over basket, transition matrix over an elastic bearing.

The matrix relations for each nod and matrix relations for each element can be written. Replacing successively, it is obtained the matrix relation between the state vectors of ends from the *n* element, $\{q\}_n^l$ and the first (point of start) $\{q\}_o^r$:

$$\{q\}_n^l = [Q]\{q\}_o^r, \tag{8}$$

where, is written

$$[Q] = [A]_n [H]_{n-1} \cdots [A]_{k+1} [H]_k [A]_{k-1} \cdots [H]_1 [A]_1.$$

The matrix [Q] is named transfer matrix for the shaft-basket system.

*Transfer Matrix for One Element of the Shaft*

The matrix relation between the state vectors of ends of an element k, $\{q\}^r_{k-1}$ and $\{q\}^l_k$ respectively, can be written:

$$\begin{Bmatrix} v_x \\ -\dfrac{\varphi_z}{\alpha_i} \\ -\dfrac{M_z}{\alpha_i^2 EI_k} \\ -\dfrac{F_x}{\alpha_i^3 EI_k} \\ v_y \\ \dfrac{F_y}{\alpha_c EA_k} \end{Bmatrix}^l_k = \begin{bmatrix} S & T & U & V & 0 & 0 \\ V & S & T & U & 0 & 0 \\ U & V & S & T & 0 & 0 \\ T & U & V & S & 0 & 0 \\ 0 & 0 & 0 & 0 & M & N \\ 0 & 0 & 0 & 0 & -N & M \end{bmatrix}_k \begin{Bmatrix} v_x \\ -\dfrac{\varphi_z}{\alpha_i} \\ -\dfrac{M_z}{\alpha_i^2 EI_k} \\ -\dfrac{F_x}{\alpha_i^3 EI_k} \\ v_y \\ \dfrac{F_y}{\alpha_c EA_k} \end{Bmatrix}^r_{k-1} . \qquad (9)$$

where: $S = [cosh(\alpha_i l_k) + cos(\alpha_i l_k)]/2$ , $T = [sinh(\alpha_i l_k) + sin(\alpha_i l_k)]/2$ , $U = [cosh(\alpha_i l_k) - cos(\alpha_i l_k)]/2$ ,

$V = [sinh(\alpha_i l_k) - sin(\alpha_i l_k)]/2$ ; $M = cos\,\alpha_c l_k$ , $N = sin\,\alpha_c l_k$ ; $\alpha_i = \sqrt{\omega}\sqrt[4]{\dfrac{\rho A_k}{EI_k}}$ ; $\alpha_c = \omega\sqrt{\dfrac{\rho}{E}}$ .

The matrix relation (9) can be written:

$$\{q\}^l_k = [A]_k \{q\}^r_{k-1} . \qquad (10)$$

The square matrix (6x6), named $[A]_k$, is the transfer matrix for the element number *k* of the shaft.

*Special Matrices*

In some nodes it is possible to appear special matrices. Between the left state vectors and right state vectors of section k, $\{q\}_k^l$ and $\{q\}_k^r$ respectively, the relation (7) can be written.

The square matrix (6x6) $[H]_k$ can be one from the matrices:

- saltus matrix, $[B]_k$, when in a node *k* appears a saltus of section. The non-zero elements of this matrix are:

$$B_{1,1} = B_{5,5} = 1;\ B_{2,2} = \frac{a_k}{b_k};\ B_{3,3} = a_k^2 b_k^2;\ B_{4,4} = a_k^3 b_k;\ B_{6,6} = a_k^4;\ \text{where } a_k = \sqrt[4]{\frac{A_k}{A_{k+1}}};\ b_k = \sqrt[4]{\frac{I_k}{I_{k+1}}} .$$

- transition matrix, $[E]_k$, over an elastic bearing in a node *k*. The non-zero elements of this matrix are:

$$E_{1,1} = E_{2,2} = E_{3,3} = E_{4,4} = E_{5,5} = E_{6,6} = 1;\ E_{4,1} = -\frac{K_x}{\alpha_i^3 EI_k};\ E_{6,5} = \frac{K_y}{\alpha_c EA_k} .$$

$K_x$ and $K_y$ are the bearing's elastic constants on the directions *Ox* and *Oy* respectively. If the elastic bearing is radial the element $E_{6,5}$ is zero. If the elastic bearing is radial-axial the element $E_{6,5}$ is non-zero.

- transition matrix, $[D]_k$, over the basket fixed in a node *k*. The non-zero elements of this matrix are:

$$D_{1,1} = D_{2,2} = D_{3,3} = D_{4,4} = D_{5,5} = D_{6,6} = 1 ;\ D_{3,1} = -\frac{m\eta\alpha_i^2}{\rho A_k} ;\ D_{3,2} = -\frac{\left(J_{Cz} + m\xi^2 + m\eta^2\right)\alpha_i^3}{\rho A_k} ;$$

$$D_{3,5} = \frac{m\xi\alpha_i^2}{\rho A_k};\ D_{4,1} = \frac{m\alpha_i}{\rho A_k};\ D_{4,2} = \frac{m\eta\alpha_i^2}{\rho A_k};\ D_{6,2} = \frac{m\xi\alpha_i\alpha_c}{\rho A_k};\ D_{6,5} = -\frac{m\alpha_c}{\rho A_k} .$$

If it take into account the gyroscopic moment, only the element $D_{3,2}$ will be modified:

- for the forward precession:

$$D_{3,2} = -\left(-J_{cy}\frac{\Omega}{\omega} + J_{cz} + m\xi^2 + m\eta^2\right)\frac{\alpha_i^3}{\rho A_k};$$

- for the backward precession:

$$D_{3,2} = -\left(J_{cy}\frac{\Omega}{\omega} + J_{cz} + m\xi^2 + m\eta^2\right)\frac{\alpha_i^3}{\rho A_k} .$$

*Determination of the Natural Frequencies*

We start from the left terminal point, *0*, and we traverse the centrifuge from an element to another till the right terminal point, *n* (fig. 3). Depending of the configuration of the centrifuge, on route, intervene the transfer matrix, [A], and the matrices [B], [D], [E] We obtain the matrix relation (8) between the state vectors, $\{q\}_n^l$ and $\{q\}_o^r$. The transfer matrix for centrifuge, written [Q], is a square matrix (6x6).

Depending of the type of leaning the boundary conditions are written and is obtained a linear and homogeneous algebraic system. A non-zero solution of this linear system exists if and only if its determinat vanishes. Therefore, a transcendental equation is obtained. By solving numerically this equation, the natural frequencies of the free vibrations are obtained.

**Application**

The method presented is applied in the concrete case of a centrifuge (fig. 4) [9], [11]. The centrifuge is formed of a shaft and a basket. The shaft has been divided in 2 elements (1 and 2). In this application the shaft of the centrifuge has not saltus of section. It has been obtained 3 nodes (0,1,2). The basket is fixed on the shaft in the node number 2. The centrifuge is elastically leaned. The bearing

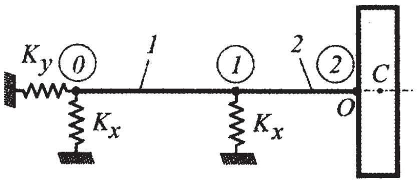


Fig. 4. Application of proposed method to a centrifuge

**Table 1**
THE FIRST EIGHT NATURAL FREQUENCIES

| Mode number | Natural frequencies [Hz] | | | | | |
|---|---|---|---|---|---|---|
| | Bending vibrations | | | Bending vibrations coupled with axial vibrations | | |
| | Backward precession | Neglected precession | Forward precession | Backward precession | Neglected precession | Forward precession |
| Mode 1 | 118.33 | 133.50 | 150.32 | 118.33 | 133.49 | 150.33 |
| Mode 2 | 393.19 | 396.75 | 401.06 | 393.19 | 396.75 | 401.05 |
| Mode 3 | - | - | - | 746.15 | 746.16 | 746.16 |
| Mode 4 | 1193.33 | 1194.06 | 1194.84 | 1193.31 | 1194.06 | 1194.85 |
| Mode 5 | 2421.78 | 2423.30 | 2424.91 | 2421.78 | 2423.37 | 2424.92 |
| Mode 6 | - | - | - | 3263.26 | 3263.26 | 3263.26 |
| Mode 7 | 3470.30 | 3475.60 | 3480.93 | 3469.92 | 3475.71 | 3480.96 |
| Mode 8 | 4685.87 | 4689.35 | 4696.31 | 4684.00 | 4692.39 | 4696.31 |

of the node 1 is radial. The bearing of the node 0 is radial-axial.

The geometrical and mechanical characteristics of the shaft-basket system are the following: lengths of elements: $l_1=0.8$, $l_2=0.05$ m; diameters of elements: $d_1=d_2=0.08$ m; mass of the basket (operating material included): m=44.94 kg; density of material and Young's modulus for shaft: $\rho=7800$ kg/m$^3$, E=21 x $10^{10}$ N/m$^2$; coordinates of the center of mass of the basket:$\xi=0.003$ $\eta$ =0.0605 m, $\zeta=0$; mechanical moments of inertia:$J_{Cy}=1.8755$ kg·m$^2$, $J_{Cz}=1.0628$ kg·m$^2$; bearing's elastic constants: $K_x = 5 . 10^{20}$ N/m; $K_y =5 . 10^{20}$N/m, revolution speed: 2000 r.p.m.

Taking into account the theory presented in Hypothesis and *Description of Mehod,* the following relations can be written:

$$\{q\}_0^r = [E]_0\{q\}_0^l;\ \{q\}_1^l = [A]_1\{q\}_0^r;\ \{q\}_1^r = [E]_1\{q\}_1^l;\ \{q\}_2^l = [A]_2\{q\}_1^r;\ \{q\}_2^r = [D]_2\{q\}_2^l .$$

Replacing successively, it is obtained:

$$\{q\}_2^r = [D]_2[A]_2[E]_1[A]_1[E]_0\{q\}_0^l \text{ or } \{q\}_2^r = [Q]\{q\}_0^l .$$

The values of the elements of the matrix [Q] depend of the angular natural frequency $\omega$

The boundary conditions are:

-in the node number 0, at the left (before the bearing): $M_z = 0$; $F_x = 0$, $F_y = 0$.

-in the node number 2, at the right (after the basket):$M_z = 0$; $F_x = 0$, $F_y = 0$

Taking into account the boundary conditions, the state vectors $\{q\}_2^r$ and $\{q\}_o^l$have the form:

$$\{q\}_2^r = \{q_{1,2} \quad q_{2,2} \quad 0 \quad 0 \quad q_{5,2} \quad 0\}_2^{r,t} \text{ and } \{q\}_0^l = \{q_{1,0} \quad q_{2,0} \quad 0 \quad 0 \quad q_{5,0} \quad 0\}_0^{l,t} .$$

Replacing, a linear and homogeneous algebraic system is obtained:

$$\begin{cases} 0 = Q_{3,1}q_{1,0} + Q_{3,2}q_{2,0} + Q_{3,5}q_{5,0} \\ 0 = Q_{4,1}q_{1,0} + Q_{4,2}q_{2,0} + Q_{4,5}q_{5,0} \\ 0 = Q_{6,1}q_{1,0} + Q_{6,2}q_{2,0} + Q_{6,5}q_{5,0} \end{cases} .$$

The condition that the system to admit non-zero solution ($\Delta = 0$) gives the transcendental equation:

$$Q_{3,1}Q_{4,2}Q_{6,5} - Q_{3,1}Q_{4,5}Q_{6,2} - Q_{3,2}Q_{4,1}Q_{6,5} + Q_{3,2}Q_{4,5}Q_{6,1} + Q_{3,5}Q_{4,1}Q_{6,2} - Q_{3,5}Q_{4,2}Q_{6,1} = 0 .$$

.By solving numerically this equation, the first eight natural frequencies have been determined.

The values have been compared with them obtained by neglecting the axial vibrations.

Using the study model in which the bending vibrations are coupled with the axial vibrations, new natural frequencies are obtained (table 1): the natural frequencies numbers 3 and 6. These values can not be obtained when the coupling of vibrations is neglected. The differences between the values obtained for the natural frequencies, neglecting coupling vibrations or not, are very small (less than 0.07%).

The objective of this paper is to discover the influence of the coupling vibrations upon the natural frequencies of the centrifuge. The objective is to discover if the coupling of vibrations do to appear new natural modes of vibrations. That is why eight values for the natural frequencies have been calculated. It is right that the values of the natural frequencies for the upper order are higher. But, the new natural frequencies number 3 can do to appear critical speed in the operating domain of the centrifuge.

Generally, the influence of the gyroscopic moment upon the natural frequencies is higher when the bending vibrations are coupled with the axial vibrations. The influence of the gyroscopic moment upon the new values of the natural frequencies, obtained by the method presented in this paper (when coupled vibrations are considered), is quasi zero (746.15 / 746.16 / 746.16 Hz and 3263.26 / 3263.26 / 3263.26 Hz).

In figure 5 are shown the relative values of the amplitudes for the first eight natural modes of vibrations.

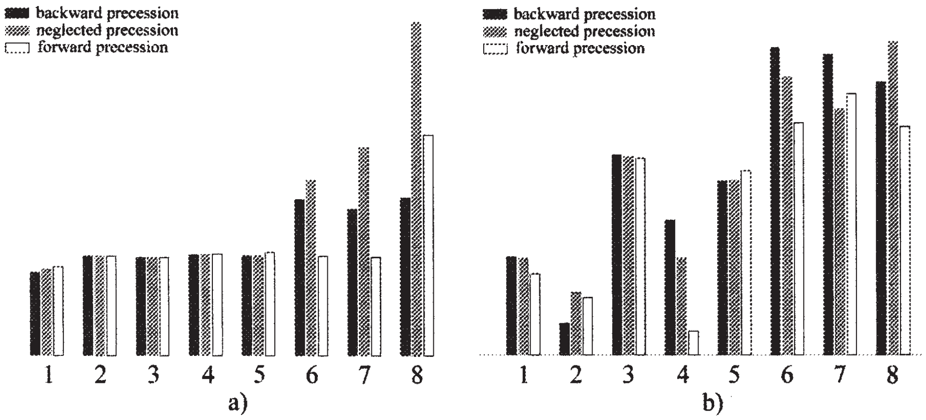


Fig. 5. The amplitudes (relative) of the vibrations: a) bending vibrations; b) axial vibrations

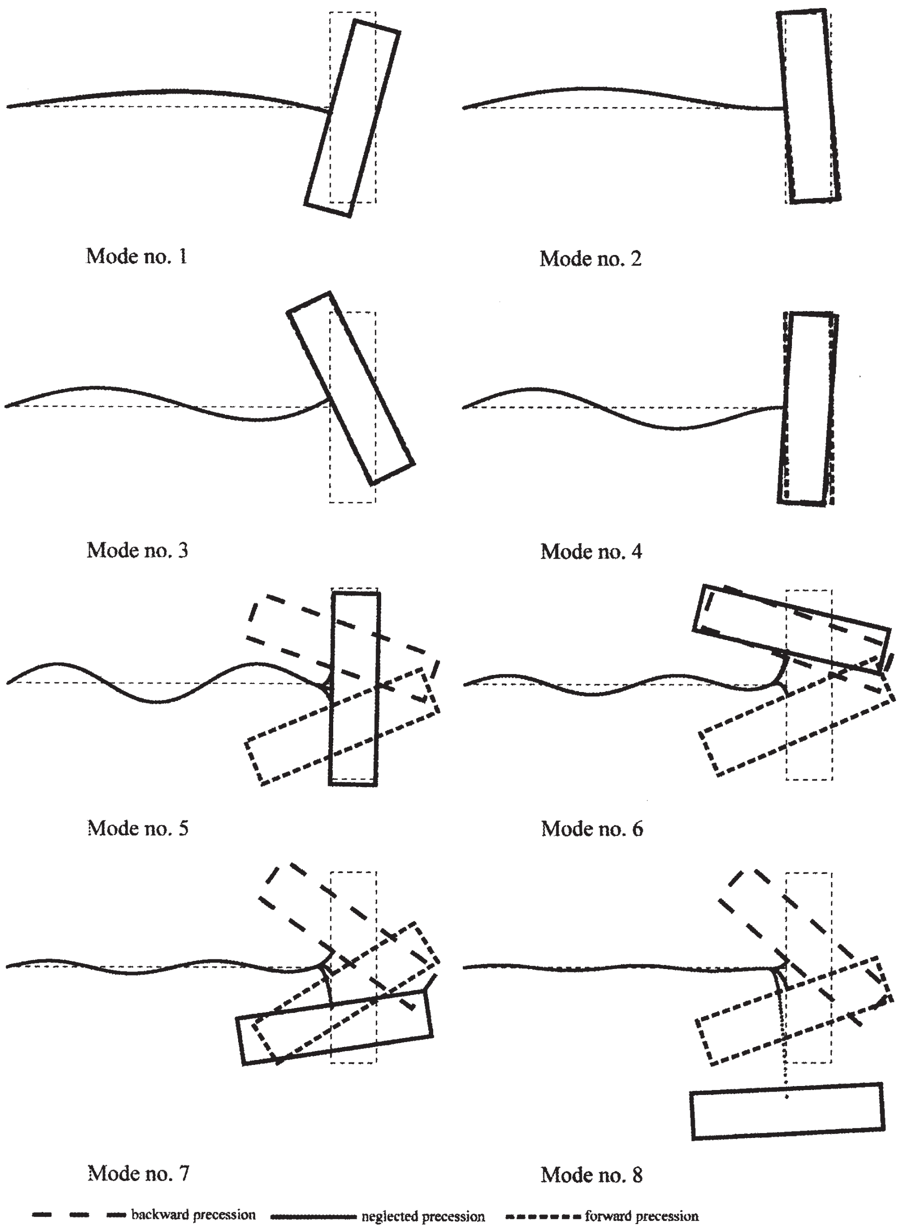


Fig. 6. The first eight natural modes of vibrations for the bending vibrations

The values have been calculated for $\frac{\varphi_z}{\alpha_i} = 1$. It is possible to see that, for the new modes of vibrations (3 and 6), the axial vibrations are not prevalent. The axial vibrations are accompanied by bending vibrations. The values of amplitudes of the bending vibrations, for the new modes of vibrations, are comparable with the values of amplitudes for the neighbouring modes of vibrations.

In figure 6 are shown the modes of vibrations for the bending vibrations. The amplitudes have been scale down or scale up in order to be possible the plotting. The deformations for the upper order are higher at the extremity of the shaft where the fixing point of the basket is. The movement of the basket is prevalent. The explanation is: since the center of mass of the basket ($C$) is not in the point of attachment to the shaft ($O$), when the accelerations are great (when the vibration frequencies are high), the deformations in the point $O$ are great. In case of the lower order the movement of the shaft is prevalent. In this case (when the vibration frequencies are

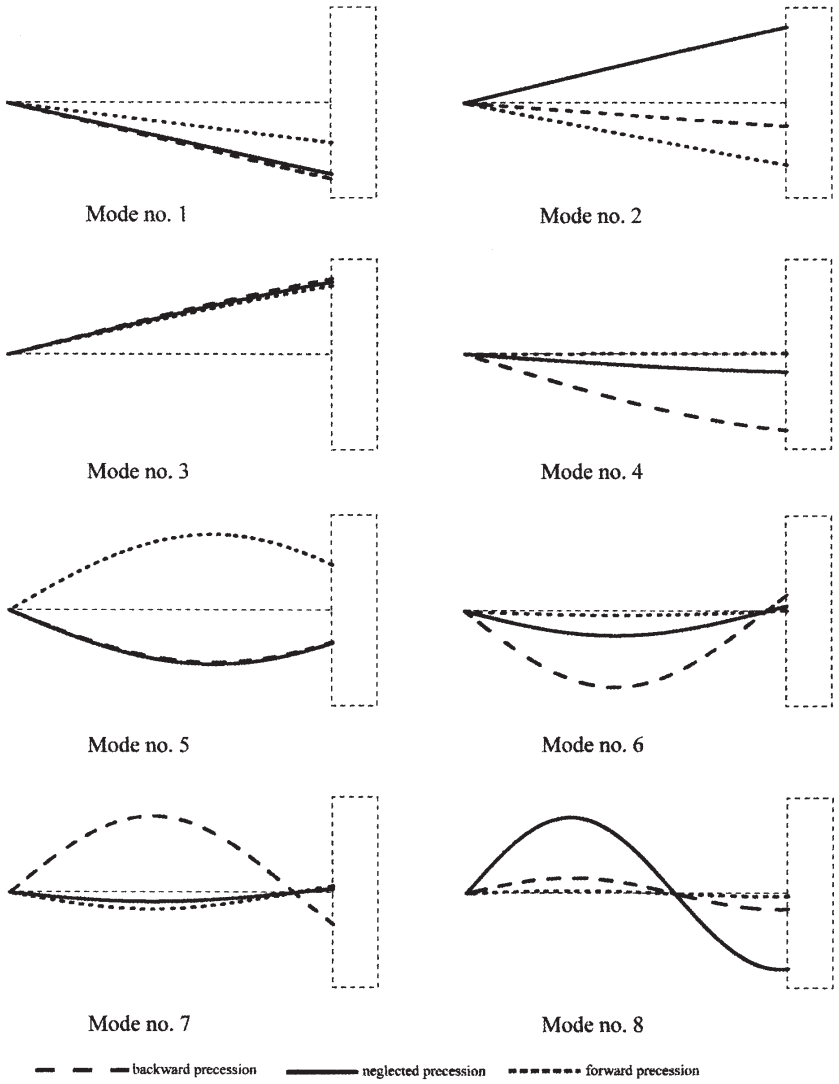


Fig. 7. Variation of the axial displacement for the first eight modes of vibrations

small), the accelerations are smaller and deformations in the point *O* are also lower.

In figure 7 are mapped the variations of the axial displacement $v_y$ for the first eight modes of vibrations. Also the amplitudes have been scale down or scale up in order to be possible the plotting. The axial vibrations are present for each mode of vibrations. From figure 6 and figure 7 it results that the bending vibrations are accompanied of axial vibrations and the study of the coupled vibrations must be do.

Because the objective of this paper was to discover the influence of the coupling vibrations upon the natural frequencies of the centrifuge, it has been chosen high values for the elastic constants of bearing. Practically, the bearings have been considered rigid.

## Conclusions

This study shown that, for a centrifuge with the basket in console that has not the centre of mass of the basket on the revolution axis of the shaft, the bending vibrations are accompanied of axial vibrations. New modes of vibrations appear. New critical speeds it is possible to appear.

The method presented in this paper, which takes in account of the coupling of the bending vibrations with the axial vibrations, can be considered more complete that the method in which the coupling of vibrations is neglected, because it gives the possibility to obtain more natural frequencies.

The proposed method is flexible and can be easily applied on different types of centrifuges.

The bearings have been considered rigid. The next steep must to be the choosing of ordinary values for the elastic constants of bearing. Also, the influence of the bearing's elastic constants upon the natural frequencies can be studied.

The influences of the mass centre position of the basket, of the constructive and operating specific features upon the natural frequencies and critical speeds can be also studied.

## Symbols

$\tau_C^{in}$ - torsor of the system of d'Alembert's fictitious forces, of the basket, in the center of mass of the basket;

$\vec{F}^{in}$ - resultant of d'Alembert's fictitious forces;

$\vec{M}_C^{in}$, $\vec{M}_o^{in}$ - resultant moment of d'Alembert's fictitious forces with respect to the center of mass, *C*, and the fixing point of the basket, *O*, respectively;

m- mass of the basket (operating material included);

$\vec{a}_C$ - acceleration of the center of mass of the basket;

$K_C$ - angular momentum about the center of mass of the basket;

$\vec{r}_C$ - position vector of the center of mass of the basket;

$\xi$, $\eta$, $\zeta$ - coordinates of the center of mass on the *Ox*, *Oy* and *Oz* axes respectively;

$F_s^n$, $F_y^{in}$, $F_z^{in}$ - projections of the resultant of d'Alembert's fictitious forces on the *Ox*, *Oy* and *Oz* axes respectively;

$M_{Ox}^{in}$, $M_{Oy}^{n}$, $M_z^{in}$ - projections of the resultant moment of d'Alembert's fictitious forces on the *Ox*, *Oy* and *Oz* axes respectively;

$v_x$, $v_z$ - displacements (deformations) on the *Ox* and *Oy* axes respectively;
$\varphi_z$ - rotation of the cross section of the shaft about the *Oz* axis;
$V_x(y)$, $V_y(y)$, $\Phi_z(y)$ - symbols for functions of *y*;
$J_{Cy}$, $J_{Cz}$ - mechanical moments of inertia with respect to *Cy* and *Cz* axes respectively;
$J_{Cxz}$, $J_{Cyz}$ - products of inertia with respect to the axes of a cartesian coordinate system having the origin in the center of mass of the basket;
$\omega$ - natural angular frequency of *n* order;
$\{q\}_k$ - state vector in the node *k*;
$[A]_k$ - transfer matrix for the element number *k*;
$\{q\}_k^l$, $\{q\}_k^r$ - state vectors at the left and right side of the node *k* respectively;
$[H]_k$ - transit matrix in the node *k*;
[Q] - transfer matrix for the shaft-basket system;
$M_z$ - bending moment;
$F_x$ - shearing force;
$F_y$ - axial force;
*S, T, U, V* - Krilov - Rayleigh functions;
*M, N* - symbols for abbreviation;
$l_k$ - length of the element number *k*;
$\rho$ - density of material of the shaft;
$A_k$ - area of cross section for the element number *k*;
*E* - Young's modulus (longitudinal modulus of elasticity);
$I_k$ - second moment of area for the element number *k*;
$[B]_k$ - saltus matrix;
$[E]_k$ - transition matrix over an elastic bearing in a node *k*;
$K_x$, $K_y$ - bearing's elastic constants on the directions *Ox* and *Oy* respectively;
$[D]_k$ - transition matrix over the basket fixed in a node *k*;
$\Omega$ - angular velocity of the shaft.

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